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# Elementary-age children's conceptions about mathematics utility and their home-based mathematics engagement

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**ABSTRACT**

Integrating multiple theoretical frameworks, the authors examined rising first- to fourth-grade students' mathematics utility conceptions—their knowledge and beliefs about the usefulness of mathematics, home-based mathematics engagement, and grade-level differences in mathematics utility conceptions and home engagement. Most children viewed mathematics as heavily focused on low-level mathematics operations and as learned and used primarily in school. Older children showed more awareness of mathematics as part of daily living, but still viewed mathematics as mostly school-based—more so than their younger counterparts. Results suggest that awareness of mathematics in daily life may be associated with children's mathematics utility value (perceived usefulness of mathematics). Although children engaged in activities at home with the potential to foster mathematics development, the frequency of engagement was not related to their awareness of mathematics in daily activities. Thus, there may be untapped opportunities for young children to connect the mathematics they learn in school to their daily life.

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Mathematics conceptions; elementary age children; mathematics education; mathematics utility value; productive disposition

Many children in the United States earn low scores on standardized mathematics assessments (National Mathematics Advisory Panel, 2008). For example, only 40% of U.S. fourth-grade students, 34% of eighth-grade students, and 25% of 12th-grade students in 2017 scored in the proficient or advanced range on the 2017 National Assessment of Educational Progress mathematics assessment (U.S. Department of Education, 2018). Given the importance of mathematics for subsequent academic and vocational success (Clark, 1988; National Mathematics Advisory Panel, 2008), it is critical to understand factors associated with children's mathematics learning, especially those outside the school context.

Research on children's mathematics learning outside of school generally focuses either on the specific mathematics activities they engage in (e.g., Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009; Niklas & Schneider, 2014) or their mathematics self-concepts (e.g., Muenks, Wigfield, & Eccles, 2018; Musu-Gillette, Wigfield, Harring, & Eccles, 2015; Schoenfeld, 1992). Far less research has focused on children's understanding of what mathematics is, how they use mathematics in daily activities, and how mathematics knowledge is acquired. In the present study we focused on children's conceptions about mathematics and their engagement in mathematics activities at home. More specifically, we addressed children's mathematics utility conceptions, that is, how much children believe that mathematics is useful and important in their lives and how they believe they acquire that knowledge. We also examined grade-level differences (rising first- through fourth-grade students) in mathematics utility conceptions and home engagement.

## Theoretical approach to mathematics utility conceptions

We conceptualize mathematics utility conceptions as a multidimensional construct formed by two dimensions: children's knowledge about mathematics and beliefs about mathematics utility. Knowledge about mathematics refers to the extent of children's knowledge of the aspects of mathematics (mathematics concepts) and the ways in which mathematics can be used by themselves and others across different contexts (applicability of mathematics). Beliefs about mathematics utility refers to children's motivational beliefs, or how they feel, about the usefulness of mathematics (utility value and productive disposition; see Figure 1 for a visual representation). An important distinction between these two dimensions is that knowledge about mathematics assesses how much children know about the breadth of mathematics and the potential uses of mathematics in various daily activities and beliefs about mathematics utility assesses value that children place on mathematics for themselves and others.

**Knowledge about mathematics.** Children's ability to assess their own knowledge of mathematics is related to their mathematics learning and achievement (Dunlosky &

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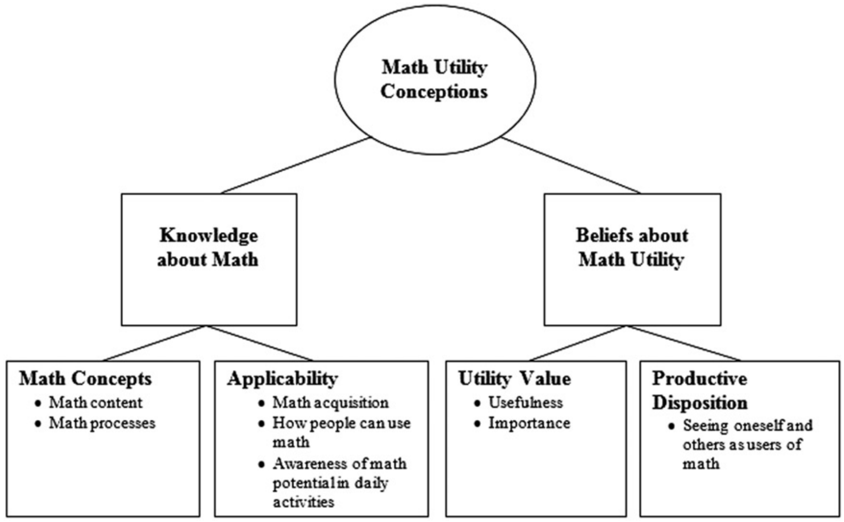


**Figure 1.** Math utility conceptions model.

Rawson, 2012; Vo, Li, Kornell, Pouget, & Cantlon, 2014). A growing body of research has examined the development of children's mathematical knowledge both in school and at home (Browning et al., 2016; Krawec, Huang, Montague, Kressler, & de Alba, 2013; Rosenzweig, Krawec, & Montague, 2011; Van Oers, 2010).

***Mathematics concepts.*** Children need to understand what mathematics is before they can develop conceptions about its usefulness. Consistent with the most recent version of the National Council of Teachers of Mathematics standards (National Council of Teachers of Mathematics, 2000), the conceptual model in this study views mathematics knowledge or concepts as consisting of content and processes. Content includes number and operations, algebra, geometry, measurement, and data analysis and probability. Processes include problem solving, reasoning and proof, communication, connections, and representations. Although these standards were published nearly two decades ago, they remain an integral part of current mathematics curricula. The National Council of Teachers of Mathematics standards, along with the National Research Council's (2001) model for developing mathematical proficiency, were the foundation for the creation of the Common Core State Standards for Mathematics (Common Core State Standards Initiative, 2010; Kendall, 2011). In addition, children's content and process knowledge build on and influence the development of one another; both are considered critical for the development of mathematics proficiency (Rittle-Johnson, 2017).

To the best of our knowledge, only two studies, Perlmutter, Bloom, Rose, and Rogers (1997) and Mazzocco, Hanich, and Noeder (2012), have investigated kindergarten through third grade children's knowledge of what mathematics is. Perlmutter et al. (1997) found that these children's definitions of mathematics consisted primarily of number and operations. Mazzocco et al. (2012) coded responses of second and third grade children's descriptions of what mathematics is using a 5-point scale that ranged from irrelevant responses to responses where children defined mathematics as a useful tool. Children's definitions included mainly basic mathematics principles or mechanics (i.e., numbers and operations). Unfortunately, coding of children's definitions of mathematics in both studies did not include mathematics concepts and processes, both of which are important for the development of mathematics knowledge (Rittle-Johnson, 2017). The present study provides a broader scope of children's definitions of mathematics by aligning coding for these responses to the National Council of Teachers of Mathematics (2000) mathematics content and process standards.

***Applicability of mathematics***. Despite the importance of children's conceptions of mathematics (De Corte & Verschaffel, 2006; Muis, 2004) and recent efforts to better understand the role of mathematics utility in children's mathematics development (Mazzocco et al., 2012; Rozek, Hyde, Svoboda, Hulleman, & Harackiewicz, 2015), we still know fairly little about children's understanding of how mathematics can be applied in different contexts. Much of the current research about knowledge of the applicability of mathematics in daily life has been conducted with high school and college students and shows that learning mathematics through "real-world" applications is positively associated with using mathematics to solve real-world problems (Barab, Squire, & Dueber, 2000; Herrington, Reeves, & Oliver, 2013; National Mathematics Advisory Panel, 2008). For example, Herrington et al. (2013) and Barab et al. (2000) found that when college students were taught in authentic learning environments (specific real-world contexts), they were better able to integrate and apply this knowledge in their daily lives. Hulleman and colleagues (Hulleman & Harackiewicz, 2009; Hulleman, Kosovich, Barron, & Daniel, 2017) found that utility value interventions designed to increase the connections that high school and college students made between course material and their lives increased how much students valued the course and their performance, especially for the lowest-performing students.

Little research has examined connections between school mathematics and mathematics in daily activities with younger children. Perlmutter et al. (1997) asked children about the usefulness of mathematics for cooking and going to the grocery store. Although children were aware of some uses for mathematics in those activities, their awareness was very limited. Children in kindergarten who were taught using the Realistic Mathematics Education curriculum, which presents mathematics problems using daily activities such as visiting the grocery store or a museum, showed significantly greater growth in early mathematics skills than children taught with the standard curriculum (Papadakis, Kalogiannakis, & Zaranis, 2017).

**Beliefs about mathematics utility.** Children's beliefs about mathematics are associated with their early mathematics skills (see De Corte & Verschaffel, 2006; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005; Wigfield et al., 2015; Wigfield, Eccles, Roeser, & Schiefele, 2008; Wigfield, Eccles, Schiefele, Roeser, & Davis-Kean, 2006), which, in turn, are related to later mathematics achievement (Duncan et al., 2007; Jordan, Glutting, & Ramineni, 2010; Watts, Duncan, Siegler, & Davis-Kean, 2014). For example, researchers have found that beliefs such as task-related academic motivation and self-concepts about performance have reciprocal and cumulative effects on future mathematics achievement (Aunola, Leskinen, & Nurmi, 2006; Marsh & Martin, 2011). In particular, beliefs about learning mathematics are generally associated with greater effort, higher self-efficacy in mathematics, and engagement in mathematical learning (Pajares & Miller, 1994; Schoenfeld, 1989; Wigfield & Meece, 1988), which are related to higher mathematics achievement. In our conceptual framework, children's beliefs about mathematics utility include utility value (the perceived usefulness of mathematics), and productive disposition (the belief that mathematics is useful and worthwhile, and that effort in mathematics pays off).

Eccles and colleagues (e.g., Eccles & Wigfield, 2002; Eccles, Wigfield, Harold, & Blumenfeld, 1993; Wigfield & Eccles, 2000) have extensively studied the link between motivation and achievement using their expectancy-value theory of motivation. Utility value is one of the two components (interest and utility value) of Eccles's subjective task value theory (Eccles & Wigfield, 2002; Eccles et al., 1993). The expectancy-value theory suggests that children's expectations of success and the value they place on academic tasks influence achievement choices, performance, effort, and persistence. In the present study, we extended previous work on the topic by considering children's knowledge and beliefs about mathematics utility. Participants were elementary school–age children, a younger age group than is typically studied. We also explored relations between children's mathematics utility conceptions and their engagement in mathematics activities at home to better understand how children's activities outside of the school context may be associated with their mathematics utility conceptions.

***Utility value.*** The majority of research examining the link between motivation and academic achievement uses Eccles's expectancy-value theory of motivation. The body of work related to the expectancy-value theory demonstrates that subjective task value, the value that one assigns to a task, including utility value, is positively related to mathematics achievement test scores, grades in mathematics, and the number and type of upper-level mathematics courses selected in high school (Guo, Marsh, Parker, Morin, & Yeung, 2015; Marsh & Martin, 2011; Marsh et al., 2005; Musu-Gillette et al., 2015; Singh, Granville, & Dika, 2002). This is particularly important, because subjective task value tends to be relatively high in early-to-late elementary school grades but declines significantly beginning around the transition to middle school (Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002). Understanding how to maintain more positive utility value beliefs is important, because positive subjective task value beliefs are associated with higher mathematics achievement, lower mathematics anxiety, and higher-level mathematics course selection for fourth- to ninth-grade students (Meece, Wigfield, & Eccles, 1990; Spinath, Spinath, Harlaar, & Plomin, 2006).

In the present study, we extended Eccles's subjective task value theory by expanding the ways in which utility value is measured with children. We focused on utility value because it is the only component within Eccles's theoretical model that specifically relates to mathematics utility conceptions and achievement (see also Mazzocco et al., 2012).

***Productive disposition***. The National Research Council (2001) recognized the importance of mathematics utility by including productive disposition, children's beliefs that they are users of mathematics, and that mathematics is useful and worthwhile, as one of their five "strands" of mathematics proficiency. A productive disposition towards mathematics is important for developing mathematics knowledge and skills (Clements, 2001; Muis, 2004). The limited research on children's beliefs about how effort and engagement in mathematics will benefit their mathematics skills focuses on beliefs about mathematical learning and problem solving (De Corte, Op't Eynde, & Verschaffel, 2002; McLeod, 1992; Schoenfeld, 1992; Tsao, 2004) and has primarily used older children and adolescents.

## Developmental changes in mathematics utility conceptions

As children progress through school, their knowledge of mathematics concepts changes (Clements & Sarama, 2014; Geary, 2006; Rittle-Johnson, 2017). They learn new mathematical operations and procedures and are exposed to new types of problems. With new exposures to mathematics in their environment, children have the potential to build new knowledge about the usefulness of mathematics in daily activities and different sources from which they can learn mathematics (Clements & Sarama, 2007; Papadakis et al., 2017; Perlmutter et al., 1997). Little research has examined grade-level differences in children's definitions of what mathematics is and whether they believe it is learned or used outside of the school context. However, research in other related mathematics conceptions shows the importance of examining developmental changes.

Several studies have shown that competence and expectancy beliefs in mathematics decline from elementary school through high school (Dweck & Elliott, 1983; Jacobs et al., 2002; King & McInerney, 2014; Muenks et al., 2018; Nagy et al., 2010). Less is known about the development of mathematics utility beliefs. Musu-Gillette et al. (2015) found that, on average, utility value was highest in Grade 4; children showed an overall decline in mathematics utility value through early college, although the rates of decline reflected group differences. However, other research suggests that mathematics utility beliefs can also improve through targeted interventions (Hulleman et al., 2017; Jansen, 2012; Mitchell, 1999). Nevertheless, the youngest children in these studies were in Grade 4, so this research does not offer information about developmental changes in early elementary school. The present study builds on prior research by examining grade-level differences in knowledge and beliefs about mathematics utility for early elementary-age children.

## Mathematics engagement at home

Children acquire mathematics knowledge from their environment even before they start school (e.g., Clements & Sarama, 2014; Elliott & Bachman, 2017; Ginsburg, Lee, & Boyd, 2008; Siegler & Mu, 2008). Nevertheless, children's engagement in mathematics is limited (Plewis, Mooney, & Creeser, 1990; Saxe, Guberman, & Gearhart, 1987). For example, Tudge and Doucet (2004) found that preschool age children from Black and White, low and middle socioeconomic status families infrequently engaged in mathematics-related activities either at home or at their child care centers. Moreover, even though children may engage in activities that have the potential to foster mathematics skills, they do not necessarily focus on mathematics when engaging in these activities. For example, even when a child plays with blocks, something which could involve mathematics, she or he may focus on the color or texture of the blocks rather than the shape or number, two potential mathematics-related components. In addition, other research has shown that even though children may engage in mathematics activities, they are likely to be involved in basic mathematics. Seo and Ginsburg (2004), for example, observed young children during free play and found that children engaged in mathematics-related talk and activities, but the complexity of their interactions was often low.

Children's limited engagement in mathematics at home is problematic given the relevance of this involvement for fostering mathematics learning. Engagement in developmentally appropriate mathematics activities at home is generally positively associated with children's early mathematics knowledge, especially for children in kindergarten and early elementary school (see reviews by Blevens-Knabe, 2016; Elliott & Bachman, 2017; Thompson, Napoli, & Purpura, 2017). Engagement in formal mathematics activities, such as completing worksheets, and informal ones, such as playing board games, positively predicts children's mathematics skills (LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; LeFevre et al., 2009; Niklas & Schneider, 2014; Ramani & Siegler, 2008, 2014; Skwarchuk, Sowinski, & LeFevre, 2014). In addition, the frequency and quality of parents' "number talk" relates to children's development of early number skills (Gunderson & Levine, 2011). For example, Levine and colleagues (Gunderson & Levine 2011; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010) found that the amount of talk about number that parents engaged in significantly predicted children's later cardinal number knowledge. Although the majority of parents' mathematics talk was labeling cardinal values or counting, talk about more advanced content, such as large number sets (4–10) with corresponding objects present, was most strongly associated with children's subsequent cardinal number knowledge. Whereas some research has examined which mathematics activities children engage in at home (LeFevre et al., 2009; 2010; Saxe et al., 1987; Sonnenschein et al., 2012; Sonnenschein, Metzger, & Thompson, 2016), little research has described how children engage in such activities. This is important because, as noted previously, whether an activity fosters mathematics skills may depend on the nature of engagement in that activity.

## Relation between children's mathematics utility conceptions and engagement

Limited research has examined the relation between children's mathematics utility conceptions, as defined in the present study, and their engagement in mathematics activities at home. Most of the research examining these constructs has focused on one dimension of mathematics conceptions (Eccles et al., 1993; Jacobs et al., 2002) or has examined the association between home engagement and achievement (LeFevre et al., 2010; Levine et al., 2010). The limited research that has examined mathematics conceptions and engagement in mathematics activities at home has shown that the quality of the home numeracy environment, indexed by measures of cognitive stimulation and home learning opportunities, positively predicts children's interest, a different aspect of subjective task value, for engaging in mathematics activities (Gottfried, Fleming, & Gottfried, 1998). Similarly, Eccles's expectancy-value theory of motivation (Eccles & Wigfield, 2002) posits that parents' beliefs about mathematics utility and the support they provide to their children with mathematics also contribute to children's utility value beliefs (Chouinard, Karsenti, & Roy, 2007; Simpkins, Fredricks, & Eccles, 2012; Wigfield & Eccles, 2000).

## The present study

In the present study, we examined rising first- to fourth-grade students' mathematics utility conceptions (knowledge about mathematics, beliefs about mathematics utility), their engagement in mathematics at home, and the association between the two.

Research Question 1: What knowledge about mathematics do children have and how does this differ across grade levels?

We examined how children define mathematics (mathematics concepts), and the extent to which they are aware of the potential uses of mathematics outside of school (applicability). Based on prior research examining children's definitions of mathematics (Mazzocco et al., 2012; Perlmutter et al., 1997), we hypothesized that children's definitions of mathematics would focus primarily on basic content knowledge, such as numbers and operations. We also hypothesized that children would think mathematics is something used primarily in school (Perlmutter et al., 1997). As children complete more grade levels in school and have more experience with mathematics in and outside of school, their knowledge about mathematics will shift to include higher-level operations, such as multiplication/division and skip counting (e.g., counting by twos or fives), and they may learn and experience more applications of mathematics in daily life.

Research Question 2: How much utility value do children place on mathematics tasks (utility value) and who do they see as users of mathematics (productive disposition)? Do these beliefs differ across grade levels?

Eccles et al.'s (1993) work suggests that young children place high value on mathematics utility. Current research does not inform hypotheses regarding young children's productive disposition, so in the present study we explored the descriptive nature of children's mathematics utility beliefs.

Research Question 3: What kinds of mathematics activities do children report engaging in most frequently at home? Does frequency of engagement vary by grade level? Additionally, do children identify mathematics-related aspects of their engagement in some activities?

There is limited research on mathematics home engagement to inform hypotheses regarding the frequency and nature of children's mathematics home engagement. Accordingly, we explored the descriptive nature of children's mathematics home engagement.

Research Question 4: What is the association between children's mathematics utility conceptions and their home mathematics engagement? Based on previous research that found associations between mathematics engagement and children's mathematics skills (e.g., LeFevre et al., 2010; Siegler & Ramani, 2009; Skwarchuk et al., 2014), we hypothesized a similar relation exists between mathematics engagement and children's conceptions, such that the frequency of children's home mathematics engagement would be positively associated with children's knowledge about mathematics (mathematics applicability) and beliefs (utility value).

The present study extends prior research in three ways. One, it adds to the very limited research on children's knowledge of how mathematics is used in real-world contexts and how that knowledge relates to mathematics engagement. Two, we examined productive disposition, an understudied construct, in children in Grades 1–4, an understudied age group. Three, we investigated associations between mathematics conceptions and children's engagement in mathematics activities at home, something we know little about because research on children's engagement has focused primarily on the association between engagement and mathematics skills.

## Method

### *Participants*

Ninety-nine children (58 boys) were recruited during the summer and early fall of 2010 and 2013 from schools and summer camps in the mid-Atlantic region. Most of the children (82%) were interviewed during the summer. Thirty-three participating children ($M$ age = 6.43 years, $SD$ = 0.39 years) were entering or had just entered Grade 1, 23 were entering or had just entered Grade 2 ($M$ age = 7.33 years, $SD$ = 0.35 years), 23 were entering or had just entered Grade 3 ($M$ age = 8.42 years, $SD$ = 0.33 years), and 20 were entering or had just entered Grade 4 ($M$ age = 9.51 years, $SD$ = 0.42 years). Children were European American/White ($n$ = 48), African American/Black ($n$ = 21), Chinese American ($n$ = 10), Hispanic/Latino ($n$ = 10), or multiracial ($n$ = 10). Five of the Hispanic/Latino children spoke primarily Spanish; the remaining five spoke primarily or only English. We did not collect specific data regarding parents' highest education level or household income, but we know that the majority of our sample was recruited from locations that serve middle income families whose parents, on average, have at least a bachelor's degree. However, about 20% of our sample was recruited from locations that serve low-income families, whose parents, on average, have not completed a college degree.

## Measures

### *Knowledge about mathematics*

**Mathematics concepts**. Children were asked "What is math?" consistent with questions from Perlmutter et al. (1997). Coding of responses was based on the National Council of Teachers of Mathematics (2000) content and process standards (i.e., numbers and operations, problem solving). See Table 1 for a list of codes and exemplary quotes for all constructs. For this and other open-ended responses, interrater reliability was established by having two raters independently code up to 50% of the responses for each item. The researchers met after coding the transcripts to review their codes and reached consensus. Intercoder reliability was tested using Cohen's kappa (Cohen, 1960). A kappa guideline of .70 was used to determine acceptable inter-rater reliability (Fleiss, 1981; Landis & Koch, 1977). If acceptable kappa levels were not reached in the first round of coding, the coding scheme was reviewed and modified, if necessary, and a new set of responses was coded. This process continued until kappas were at least .70 for every coding category. Final kappas ranged from .70 to 1.00 for each code within each construct unless otherwise noted. Remaining responses were then coded by one of the raters who had reached acceptable reliability.

**Applicability of mathematics.** We used three open-ended questions adapted from Perlmutter et al. (1997) to examine this construct (see Table 1 for a description of codes). The first question measured how children believe mathematics knowledge is acquired. Children were asked "How do you learn math?"

Children also were asked, "How does {person[s] mentioned} use math?" for each specific person that the child first mentioned used mathematics (this was a follow-up question, which is discussed further under productive disposition: "Who uses math?"

The third question asked whether and how children believed that mathematics was used in 10 activities: playing

**Table 1.** Description of measures.

| Constructs | Item | Codes or scale |
|---|---|---|
| Knowledge about mathematics | | |
| Mathematics concepts | "What is math?" | Content<br>Number and operations ("math is numbers")<br>Algebra ("like when 7 minus X equals 2")<br>Geometry ("math is shapes")<br>Measurement ("we use rulers to see how many inches the eraser is")<br>Data analysis and probability ("when you make charts with everyone's eye color in the class")<br>Processes<br>Problem solving ("math is solving problems")<br>Reasoning and proof ("logic is a part of math")<br>Communication ("when the teacher tells us to tell how we got our answer")<br>Connections ("math is in science and physics too")<br>Representations ("we use base 10 blocks to find the answers") |
| Applicability of mathematics | "How do you learn math?" | School ("I learn at school," "from my teacher," or "by doing your homework")<br>Parents ("with my daddy," "my parents help me with my homework," or "playing math games on the computer with mommy")<br>Learning math by doing it ("you have to practice it")<br>Other activities that are not specific to home or school ("we learn math by using blocks")<br>Friends/siblings ("my sister helps me learn") |
| | "How does {person} use math?" | School-related uses ("they teach students how to do math")<br>Home-related uses ("they play with math games at home")<br>Math operations ("they add and subtract things")<br>Job-related math operations ("she adds things when she's at work")<br>Daily living math operations ("he measures cups when he's cooking") |
| | "Do you think that math is used when you do (activity)? How is math used in (activity)?" | 0 Child did not identify that math was used or if she or he said that math was used, but the description was not related to math ("you read the instructions on the card").<br>1 Child said that math was used in the activity, but did not describe how ("I don't know").<br>2 Child described a basic math skill, such as number recognition ("when you play cards, there are numbers on them") or operation, such as counting ("you count the spaces when you roll the die")<br>3 Child described an advanced math skill, such as number magnitude comparison ("when you keep score, you have to know whose score is bigger so you know who won") or measurement ("for the recipe, you have to measure 1/4 cup of flour and 2 cups of sugar") |
| Beliefs about mathematics | | |
| Utility value | "Math is useful outside of class." "I need to learn math to do well in school." "It is important for me to learn math." "My parents think it is important for me to learn math." "I think it is important for everyone to learn math." "It is important for me to do well in math." | 1 "Not at all like me"<br>2 "A little like me"<br>3 "A lot like me" |
| Productive disposition | "Who uses math?" | Teachers Parents Children Other adults Everyone |
| Home mathematics engagement | | |
| | How often do you (activity) at home? (Example activities: play board games, play card games, help with cooking) | 1 "Almost never"<br>2 "Sometimes"<br>3 "Almost every day" |

board games, card games, and video games; doing puzzles; cooking; helping at the grocery store; building with blocks or Legos; using or playing with money; using maps or a globe; and keeping score in games or sports. Children first were asked, "Some children think math is used when they [play board games], some think math is not used at all. Do you think math is used when you play board games?" If children responded "yes," they were then asked how mathematics was used in the activity. A child's response was coded on a 4-point scale: a score of 0 if she or he did not identify that mathematics was used or if she or he said that mathematics was used but the description was not related to mathematics (e.g., "when cooking, you read the words on the page"); a score of 1 if the child said that mathematics was used in the activity, but did not elaborate or articulate about how; a score of 2 if she or he described a basic mathematics skill; and a score of 3 if she or he described an advanced mathematics skill. Interrater reliability was established by having two raters independently code about 50% of the responses. Because of the meaningful differences between scale values, we wanted to be sure that independent coders were in complete agreement before moving forward; therefore, 100% exact agreement was reached before coding the remaining responses. A composite was created by averaging scores for each activity examined. Cronbach's alpha for the applicability scale was .84.

### Beliefs about mathematics utility

**Utility value.** We used six items to create a utility value measure (see Table 1). Items were adapted from measures used to grasp mathematics or reading motivation (Baker & Scher, 2002; Eccles et al., 1993; Sonnenschein, Baker, & Garrett, 2004; Wigfield & Guthrie, 1997). Examples of items include "Math is useful outside of school" and "It is important for me to learn math." Children were asked to report whether they felt each item was "not at all like me," "a little like me," or "a lot like me." Three nonmathematics activities were presented as examples at the outset in order to familiarize children with the rating scale. A composite was created by averaging the scores on the six items. Cronbach's alpha for the utility value measure was .69.

**Productive disposition.** To measure the extent to which children see themselves and others as users of mathematics, we used Perlmutter et al.'s (1997) question, "Who uses math?" (see Table 1). Responses were categorized as teachers, parents, children, and other adults (most commonly mentioned other adults were scientists, architects, accountants, and adult relatives). An additional category was coded if a child said that everyone does mathematics. Final kappas for each coding category were 1.00.

**Mathematics engagement at home.** An index for child-reported frequency of mathematics home engagement was created by averaging frequency of engagement in 13 mathematics activities at home, including playing board games, playing video games, helping with cooking, helping at the grocery store, and building with blocks or Legos (see Table 1). These items were adapted from other measures of children's frequency of mathematics engagement at home (Sonnenschein et al., 2012). Response options were "almost never," "sometimes," and "almost every day." Three nonmathematics activities were presented as examples to familiarize children with the rating scale. Based on results from pilot testing, the rising first-grade students received an abbreviated version of the mathematics engagement measure. It did not include four activities: keeping score in games or sports, playing with or using money, using maps or globes, and using a calculator. Cronbach's alpha for the frequency of engagement scale was .66 for all 13 items and .50 for the nine items common to all children. The less-than-optimal alpha values likely reflect that a child's engagement in one activity does not necessarily mean she or he will engage in another activity.

**Demographic information**. As part of the consent documents, parents were asked to provide their child's age, grade level in school in the fall, gender, and race or ethnicity (African American/Black, European American/White, Hispanic/Latino, Asian/Pacific Islander, or "other").

## Procedure

Children were interviewed individually by a trained graduate or an advanced undergraduate research assistant. Each interview took place in an empty room in the child's home or summer camp or school. Sessions lasted 15–20 minutes and were recorded. The interviewer also took notes of the child's responses. Children were interviewed in their preferred language which was English for all but five of the Latino/a children. Those five children were interviewed in Spanish by a native Spanish speaker. Interviews conducted in Spanish were transcribed in Spanish, then translated into English, and then back-translated to ensure accuracy.

## Analytic plan

The mathematics utility conceptions model presented in this article (see Figure 1) is the conceptual model that guided this study. We examine the components individually and some relations among them, but do not statistically test the model itself. We use a quantitative approach, described within each of the results subsections, to address the research questions. We complement these quantitative findings, as appropriate, with illustrative quotes from participants.

**Length of utterance.** We completed a length-of-utterance analysis for each open-ended item to control for potential developmental differences in the length of children's responses as well as the possibility that children who simply speak more words may articulate more about their mathematics conceptions. Similar to how length of open-ended responses was assessed in related research (e.g., Denscombe, 2008; Wang, 2004), for each response, we counted the number of words the child used, with the exception of filler utterances such as "um" and "uh."

Preliminary analyses showed that there were significant length of utterance differences across grade levels for some items. Accordingly, for analyses examining differences between each grade level for open-ended item responses, we controlled for children's length of utterance for that response. We conducted analyses with and without controlling for length of utterance; however, the pattern of results was very similar. In what follows, we only report analyses controlling for length of utterance.

## Results

### Mathematics utility conceptions

***Knowledge about mathematics.*** Analyses for open-ended items of mathematics concepts and applicability of mathematics were coded and analyzed descriptively. Depending on the nature of the dependent variable, dichotomous or continuous, we used logistic regressions or analyses of covariance to examine grade-level differences in responses. Analyses of covariance with grade level as the between-subjects factor and length of utterance as the covariate were used to examine grade-level differences in scale scores and number of activities that children identified as featuring mathematics. Fisher's least significant difference post hoc tests were used to examine differences between specific grade levels.

**Table 2.** Responses to "What is math?"

| Coding category | Overall $N = 99$ | Rising first-grade student $n = 33$ | Rising second-grade student $n = 23$ | Rising third-grade student $n = 23$ | Rising fourth-grade student $n = 20$ |
|---|---|---|---|---|---|
| Content | 91.9% | 84.8% | 91.3% | 95.7% | 100.0% |
| Number and operations | 89.9% | 81.8% | 91.3% | 95.7% | 95.0% |
| Counting | 18.2% | 24.2% | 30.4% | 4.3% | 10.0% |
| Number knowledge | 2.0% | 0.0% | 4.3% | 4.3% | 0.0% |
| Number transformations | 66.7% | 45.5% | 65.2% | 82.6% | 85.0% |
| Number patterns | 6.1% | 17.4% | 30.0% | 0.0% | 0.0% |
| Algebra | 3.0% | 0.0% | 0.0% | 4.3% | 10.0% |
| Geometry | 8.1% | 12.1% | 8.7% | 0.0% | 10.0% |
| Measurement | 7.1% | 6.1% | 8.7% | 4.3% | 10.0% |
| Processes | 12.1% | 6.1% | 4.3% | 21.7% | 20.0% |
| Problem solving | 5.1% | 0.0% | 4.3% | 8.7% | 10.0% |
| Connections | 5.1% | 3.0% | 0.0% | 8.7% | 10.0% |

*Note*. Children were able to give more than one response; therefore, percentages do not total 100%.

**Table 3.** Responses to "How do you learn math?"

| | Overall $N = 99$ | Rising first-grade student $n = 33$ | Rising second-grade student $n = 23$ | Rising third-grade student $n = 23$ | Rising fourth-grade student $n = 20$ |
|---|---|---|---|---|---|
| School | 73.7% | 51.5% | 65.2% | 100.0% | 90.0% |
| Teachers | 54.5% | 39.4% | 52.2% | 65.2% | 70.0% |
| Homework | 5.1% | 3.0% | 0.0% | 8.7% | 10.0% |
| Parents | 27.3% | 24.2% | 17.4% | 39.1% | 30.0% |
| School work | 1.0% | 0.0% | 0.0% | 4.3% | 0.0% |
| Home activities | 2.0% | 0.0% | 0.0% | 8.7% | 0.0% |
| By doing mathematics | 8.1% | 6.1% | 4.3% | 13.0% | 10.0% |
| Activities (not home/school) | 12.1% | 18.2% | 17.4% | 4.3% | 5.0% |
| Friends/siblings | 5.1% | 0.0% | 0.0% | 8.7% | 15.0% |

*Note*. Children were able to give more than one response; therefore, percentages do not total 100%.

***Mathematics concepts.*** As hypothesized, children's definitions of mathematics indicated a view of mathematics that was heavily focused on numbers and operations (see Table 2). Ninety percent of children defined mathematics as some form of numbers and operations. Most children stated that mathematics is calculations (67%, number transformations like addition and multiplication) and counting (18%). For example, children often gave responses like, "[Math is] something when you learn about numbers and how to add them up" or they gave more elaborate descriptions of operations, "[Math is] something that you learn about numbers and … when the teacher says two plus two, you say it's four and four and four equals and she snaps her fingers and we will all say eight."

A few children (12%) mentioned mathematics processes, including problem solving ("you use math to figure out difficult problems") and connections ("[Math is] something to help you go along the way because math is in a lot of things, in science, geometry, even art" or "Math is this thing with numbers and like everything, and when I say everything, I mean everything, involves math"). Children mentioned a mean of 1.33 different categories of mathematics ($SD = 0.77$; range $= 0$–$4$), which indicates that, on average, children's knowledge of the breadth of mathematics concepts is somewhat limited.

As hypothesized, there was a difference across grade levels in children's knowledge of mathematics concepts. Controlling for length of utterance, the odds that children mentioned calculations (number transformations) ($B = 0.71$, odds ratio [OR] $= 2.03$, $p = .001$) approximately doubled for each grade-level increase. This suggests that children may define mathematics by what they are doing in school, given that younger children's number transformations were typically addition and subtraction and older children's were multiplication and division. There were not significant differences across grade level for other coding categories within mathematics concepts.

***Applicability of mathematics.*** As hypothesized, children viewed mathematics as something that is learned and used in school (see Table 3). When asked how they learn mathematics, children mentioned school (74%) and learning from teachers (55%) more often than learning from parents (27%) or non–school-related activities (12%). Sixty-four percent of children mentioned only one way to learn mathematics; 28% mentioned more than one way. For example, two different responses coded as learning at school were "I learned it from school when I was in first grade." and "you have to go to school to learn math." Two responses coded as learning mathematics from teachers were, "well the teacher teaches us the subject and we have to write it down … we get a quiz to do all the things like if I was on multiplication that had to put multiplication facts" and "teachers show us like how to add and subtract."

As hypothesized, there were differences across grade levels in children's knowledge of the ways in which mathematics can be acquired. Controlling for length of utterance, the odds that children mentioned that mathematics is learned in school ($B = 1.00$, $\text{OR} = 2.72$, $p < .001$) and that mathematics is acquired with the help of a teacher ($B = 0.43$, $\text{OR} = 1.53$, $p = .026$) increased with each additional grade level. Also, the odds that children mentioned friends or siblings (generally in the context of helping with school work) increased with each additional grade level ($B = 1.55$, $\text{OR} = 4.71$, $p = .047$).

**Table 4.** Responses to "How does {person mentioned} use math?" by grade and use.

| | Overall $N = 99$ | Rising first-grade student $n = 33$ | Rising second-grade student $n = 23$ | Rising third-grade student $n = 23$ | Rising fourth-grade student $n = 20$ |
|---|---|---|---|---|---|
| School | 35.4% | 32.1% | 46.3% | 33.3% | 30.3% |
| Home | 0.6% | 0.0% | 2.4% | 0.0% | 0.0% |
| Mathematics operations | 61.8% | 53.6% | 46.3% | 72.9% | 78.8% |
| Job-related | 24.2% | 8.9% | 14.6% | 41.7% | 36.4% |
| Daily living | 5.1% | 0.0% | 2.4% | 10.4% | 9.1% |

*Note*. Children were able to give more than one response; therefore, percentages do not total 100%.

**Table 5.** Responses to "How does {person mentioned} use math?" by person mentioned and use.

| | Teachers | Parents | Children | Other adults | Everyone |
|---|---|---|---|---|---|
| School | 76.2% | 13.3% | 39.3% | 6.5% | 14.3% |
| Home | 0.0% | 0.0% | 1.6% | 0.0% | 0.0% |
| Mathematics activities | 33.3% | 60.0% | 54.1% | 89.1% | 92.9% |
| Job-related | 9.5% | 33.3% | 0.0% | 69.6% | 14.3% |
| Daily living | 0.0% | 13.3% | 4.9% | 2.2% | 21.4% |

*Note*. Children were able to give more than one response; therefore, percentages do not total 100%.

When asked the ways in which different people use mathematics, children primarily mentioned mathematics operations (62%; e.g., addition or subtraction, multiplication or division) and school-related uses of mathematics (35%) rather than home-related uses (< 1%; see Table 4). Of the children who mentioned teachers as people who use mathematics ($n = 43$), 76% reported that teachers primarily use mathematics in the school context for teaching children rather than in their daily lives outside of school ("to teach their students to learn" or "well she does let us make a one hundred chart and she lets us use digi blocks"). In contrast, children were aware of parents (33% of $n = 18$) and other adults (70% of $n = 38$) typically using mathematics for job-related activities (see Table 5). For example, one child shared, "[My parents use math] at work" or "she helps people like if they want to go on a cruise then she has to like take this much money and add it up to this much money." Another child mentioned, "scientists use it for like chemicals and stuff like one fourth of something" or "cooks measure things out."

Children's awareness of mathematics in daily activities was measured by how well children were able to identify and articulate the mathematics potential in several every day activities. On average, children were able to identify that mathematics was used in a specific activity but did not or could not fully describe how it was used ($M = 1.26$, $SD = 0.79$; range = 0.0–2.90 of possible 3). As hypothesized, controlling for length of utterance, children's knowledge of the applicability of mathematics in daily activities increased significantly across grade level, $F(3, 94) = 13.25$, $p < .001$, $\eta^2_p = .297$. Third-grade ($M = 1.84$) and fourth-grade ($M = 1.90$) students' knowledge did not differ significantly ($p = .918$). Both third- and fourth-grade students had significantly higher mathematics awareness scores than did first-grade ($M = 0.73$; $p < .001$ and $< .001$, respectively) and second-grade ($M = 0.90$; $p < .001$ and $< .001$, respectively) students who did not differ from each other ($p = 285$).

Although, on average, children's knowledge of the applicability of mathematics was limited, most children were able to articulate their awareness of how mathematics is used in some individual activities but not others. Typical examples of children identifying mathematics with a basic mathematics concept (score of 2) included "Well sometimes in dice games, you need to count the number on the dice" or "The cards have numbers on them" or "When you check out- … you gotta count the money." Typical examples of children identifying mathematics with an advanced mathematics concept (score of 3) included "fractions like … half a cup of sugar or a quart of water" or "If our team had 7 and the other team had like 3, then our team would have four more points" or "[about playing with puzzles] You have to get the right pieces in the perfect size where it has to be."

**Beliefs about mathematics.** For utility value, descriptive statistics are presented, and analyses of variance used to examine grade-level differences. We did not control for length of utterance in these comparisons, because they were scale rather than open-ended items. For productive disposition, descriptive analyses are presented, and logistic regressions used to determine grade-level differences in responses to each coding category. Because of grade-level differences, relations between the mathematics awareness and utility value scores were examined using partial correlations, controlling for grade level.

***Utility value.*** As hypothesized, in general, children believe that mathematics is useful ($M = 2.58$, $SD = 0.41$, on a scale of 1–3). Utility value scores differed across grade level, $F(3, 95) = 4.15$, $p = .008$, $\eta^2_p = .116$. Third-grade ($M = 2.70$) and fourth-grade ($M = 2.76$) students' usefulness scores were comparable to each other ($p = .600$) and significantly higher than first-grade students' ($M = 2.41$; $p = .009$ and .002, respectively). First- and second-grade students' ($M = 2.54$) scores did not differ significantly ($p = 252$).

***Productive disposition.*** Consistent with children's conceptions that mathematics is primarily school-based, children viewed mathematics as used primarily by their teachers or their peers, rather than their parents or themselves (see Table 6). When asked who uses mathematics, a higher percentage of children mentioned children, classmates, or siblings (53%); teachers (43%); and other adults (38%) than they did parents (18%).Controlling for length of utterance, for each additional grade level, the odds of children mentioning that other adults (e.g., scientists, engineers, architects; $B = 0.54$, $OR = 1.71$, $p = .007$) use mathematics or that everyone ($B = 0.54$, $OR = 1.72$, $p = .035$) uses mathematics increased. There were no significant age differences for any other category.

Controlling for children's grade level, children's knowledge about mathematics applicability was significantly related to their mathematics utility scores, $r(94) = .28$, $p = .005$. The more aware children are that mathematics

**Table 6.** Responses to "Who uses math?" by grade.

| Coding category | Overall $N = 99$ | Rising first-grade student $n = 33$ | Rising second-grade student $n = 23$ | Rising third-grade student $n = 23$ | Rising fourth-grade student $n = 20$ |
|---|---|---|---|---|---|
| Teachers | 43.4% | 42.4% | 69.6% | 34.8% | 25.0% |
| Parents | 18.2% | 21.2% | 8.7% | 26.1% | 15.0% |
| Children | 52.5% | 63.6% | 56.5% | 43.5% | 40.0% |
| Other adults[a] | 38.4% | 18.2% | 34.8% | 60.9% | 50.0% |
| Everyone | 16.2% | 9.1% | 8.7% | 21.7% | 30.0% |

*Note*. Children were able to give more than one response, so percentages do not total 100%.
[a]Other adults include scientists, architects, accountants, adult relatives, cashiers, engineers, and mathematicians.

features into their daily activities, the more strongly they believe that mathematics is useful and important.

## Home mathematics engagement

Descriptive analyses for the frequency scale of children's home mathematics engagement categories are presented. Because of grade-level differences in mathematics utility conceptions, partial correlations, controlling for grade level, examined whether home engagement was associated with mathematics applicability and mathematics utility scores.

Children, on average, reported "sometimes" engaging in mathematics activities at home ($M = 1.84$, $SD = 0.30$; range = 1.38–2.92). Thirty-five percent of children reported engaging in mathematics activities at home almost every day, and 50% reported engaging sometimes. The activities which children reported engaging in most frequently were using a computer ($M = 2.11$ of 3), playing video games ($M = 2.09$), keeping score in games ($M = 2.09$), building with blocks or Legos ($M = 2.05$), and helping at the grocery store ($M = 2.04$). The activities which children reported engaging in least frequently were using maps or globes ($M = 1.41$), using a calculator ($M = 1.50$), and playing with or using money ($M = 1.69$). Mean frequency of engagement did not differ significantly across grade level, $F(3, 95) = 2.07$, $p = .109$.

**Association between children's mathematics utility conceptions and home mathematics engagement**. To determine how children's mathematics utility conceptions were associated with children's engagement in home-based mathematics activities, we examined whether components of mathematics utility conceptions were related to the frequency of mathematics engagement. Contrary to our hypothesis, after controlling for grade level and length of utterance, children's overall frequency of engagement was not significantly associated with their overall mathematics awareness scale scores, $r(93) = -.13$, $p = .215$. Also contrary to our hypothesis, frequency of engagement was not significantly associated with children's mathematics utility scores, $r(97) = -.03$, $p = .796$, or any individual item within the utility value scale ($p > .05$).

We examined, for each activity, whether frequency of engagement was related to awareness of mathematics scores for that activity, controlling for grade level and length of utterance. Again, contrary to our hypothesis, after controlling for grade level and length of utterance, children's engagement in a specific activity was not associated with their awareness of mathematics in that activity ($p > .05$), except for playing video games, $r(94) = .21$, $p = .038$. Finally, we examined whether children's awareness of mathematics in one activity was related to their awareness of mathematics in other activities. After controlling for grade level and length of utterance, children's awareness of mathematics in a given activity was rarely significantly correlated (with a few exceptions) with awareness of mathematics in other activities (see Table 7). The lack of significant correlations suggests that even when children are aware of mathematics in one activity, they may not be able to generalize that knowledge to other activities as well.

**Table 7.** Correlations between children's mathematics applicability scores for various activities, controlling for grade and length of utterance.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1. Board games | – | – | – | – | – | – | – | – | – |
| 2. Cooking | .11 | – | – | – | – | – | – | – | – |
| 3. Grocery store | .03 | .16 | – | – | – | – | – | – | – |
| 4. Keep score | .28* | .33** | .02 | – | – | – | – | – | – |
| 5. Playing cards | –.04 | .20† | .10 | –.05 | – | – | – | – | – |
| 6. Blocks/Legos | .14 | .24* | .05 | .09 | .35*** | – | – | – | – |
| 7. Video games | .15 | .09 | .09 | .15 | .06 | .05 | – | – | – |
| 8. Money | .15 | .29* | .06 | .32** | .19† | .10 | –.07 | – | – |
| 9. Puzzles | .14 | .06 | .17 | .13 | –.04 | .17† | .14 | –.06 | – |
| 10. Maps | .01 | .16 | .21† | .11 | < .01 | .14 | <.01 | .20† | .17 |

*Note*. †$p \le .10$.
*$p < .05$.
**$p < .01$.
***$p \le .001$.

## Discussion

This study examined children's knowledge and beliefs about mathematics and its utility, and the relation between such knowledge and beliefs and their engagement in mathematics activities at home. This was one of the first studies to examine this topic with rising first- to fourth-grade students. Understanding children's mathematics utility conceptions is important for getting a more complete picture of their mathematics knowledge and beliefs. Building this understanding is important, because research shows that children's knowledge and beliefs about mathematics are associated with their mathematics achievement (e.g., Hulleman et al., 2017; Mazzocco et al., 2012; Papadakis et al., 2017; Rittle-Johnson, 2017). Three findings were of particular interest.

First, consistent with Perlmutter et al. (1997) and Mazzocco et al., (2012), children's views about what mathematics is were heavily focused on low-level mathematics operations, such as counting and number transformations, and as something learned and used primarily in school. Most children conceptualized mathematics as school-based; they displayed limited knowledge of how mathematics features in their daily lives outside of school. In other words, they did not seem to associate the mathematics they learned in school with the mathematics they may use in their everyday activities outside of school. It could be that to foster connections between mathematics learned at school and mathematics embedded in their daily lives, these connections must be made explicit. School seems to be a natural place for explicit discussions about mathematics to occur, as what children are doing at home does not appear to be sufficient for developing constructive conceptions about mathematics utility. Helping children see the relations between school mathematics and the mathematics they are using in their own daily activities may be a way for them to develop a sense of themselves as mathematics users. This is important because seeing oneself as a user of mathematics is an integral part of developing mathematics proficiency (National Research Council, 2001).

Second, there were differences in children's knowledge of mathematics across grade levels, even after controlling for what could be differences in aspects of language skills. Older children viewed mathematics as more school-based than did younger children. Specifically, older children conceptualized mathematics as number transformations that likely coincide with the higher-level operations they are learning in school and that mathematics is learned mostly at school, with the help of teachers and classmates or peers. On the other hand, older children reported higher mathematics utility value and were able to identify more ways in which people use mathematics in their daily lives outside of the school. However, even though children's mathematics awareness increased with grade, older children were still often unable to identify how mathematics is used in daily activities, indicating that there may be limitations to their knowledge of mathematics applicability at this age. There are multiple factors that may contribute to these differences across grade levels. Children's experiences at school and home provide them with increased mathematics language and knowledge, which can improve the metacognitive skills needed for mathematics development (Ginsburg et al., 2008). These experiences could include formal mathematics lessons, exposure to adults who model mathematics uses or discuss how they use mathematics in their lives, and children's own mathematics use in their daily activities.

Third, regardless of child's grade level, the frequency of engaging in mathematics activities at home was not associated with knowledge about mathematics applicability in those activities. Mathematics home engagement was also not associated with beliefs about the utility of mathematics. One reason may be that children are not labeling their activities as "mathematics." Labeling these activities as mathematics is important, because research shows that children's mathematics language is associated with their ability to recognize and communicate about their mathematics learning and well as their mathematics achievement (Bay-Williams & Livers, 2009; Purpura & Reid, 2016; Rothman & Cohen, 1989). Children's understanding of their own mathematics learning facilitates their understanding of mathematics concepts and connections (National Council of Teachers of Mathematics, 2000). Thus, it seems particularly important in the development of mathematics utility conceptions that children be knowledgeable about the potential ways that mathematics features into their daily activities and be able to apply the label of "mathematics" to those activities. They may be less likely to consider it an activity in which mathematics is useful if they do not provide such a label. Children may still learn mathematics skills through engaging in mathematics activities, but may be less likely to develop mathematics utility conceptions from those activities if they do not label the activity as "mathematics."

Another reason for the lack of relation between engagement and mathematics conceptions may be because, although children were exposed to experiences in their homes that could enable them to acquire mathematics skills (Ginsburg, Duch, Ertle, & Noble, 2012; Ginsburg et al., 2008; Sarama & Clements, 2006, 2007; Siegler & Mu, 2008), the nature of their engagement may not facilitate learning if they are not engaged in mathematics-related aspects of such activities (Seo & Ginsburg, 2004). When we asked children to describe how they engaged in two mathematics activities (helping with cooking and helping at the grocery store), we found that only 18% of children who helped with cooking and 8% who helped at the grocery store mentioned doing anything related to mathematics while engaging in that activity. Most children reported non–mathematics-related engagement when explaining what they do while cooking and helping at the grocery store including gathering ingredients ("I get my mom the meat" or "getting the things my mom wants or the bags, putting stuff in bags"), mixing or pouring ingredients ("well, I help my mom to make soup and I put the soup in the saucepan and I wash the potatoes"), and reading instructions or shopping lists ("I read the words on the page" or "my mom hands me the list and then I read it so I know what to pick out"). Thus, despite children reporting that they were frequently involved in these activities (55% helped with cooking and 77% helped at the grocery store), their participation may not be fostering mathematics learning. Although the majority of children in this study reported engaging in activities that could foster mathematics skills, few reported engaging in aspects of those activities that actually involved mathematics. These findings suggest that parents and teachers may increase children's mathematics knowledge by actively modeling mathematics-related behaviors or mathematics language, engaging their children in mathematics-related aspects of common daily life activities (e.g., cooking, grocery shopping), and by making sure their children engage in a variety of mathematics activities (Sonnenschein et al., 2016).

## Implications for practice

Eliminating mathematics educational disadvantages is an important national priority in mathematics education policy (National Mathematics Advisory Panel, 2008). Large-scale international tests, completed by fourth-, eighth-, and 12th-grade students, show deficits for children in the United States as early as Grade 4 (U.S. Department of Education, 2013, 2015a, 2015b, 2018). The Standards for Mathematical

Practice within the Common Core curriculum call for children to learn how to connect the mathematics they are learning in school with mathematics they need to solve everyday problems (Common Core State Standards Initiative, 2010; Kendall, 2011). Teachers can do this by embedding problems in everyday situations and explicitly connecting mathematics learning to daily activities.

Another avenue for improving children's mathematics conceptions is focusing on home-based opportunities. Parents can demonstrate, by modeling or explicitly discussing with children, the ways in which they use mathematics in their daily lives (e.g., paying bills, cooking, counting money at the grocery store) for children to recognize their parents as users of mathematics. The extent to which parents engage in number talk at home relates to children's number knowledge (Blevins-Knabe & Musun-Miller, 1996; Gunderson & Levine, 2011; LeFevre et al., 2009; Levine et al., 2010). Discussion of mathematics utility value may have a similar impact on children's mathematics utility conceptions. There also may be ways to add mathematics utility to these informal discussions to increase children's knowledge about the applicability of mathematics and develop positive beliefs about the usefulness of mathematics. As Levine and colleagues (Gunderson & Levine, 2011; Levine et al., 2010) have shown with other aspects of mathematics talk, parents' discussions of mathematics are related to their children's knowledge. However, to help parents do this, we must better understand the ways in which their conceptions impact the nature of children's engagement in mathematics activities at home. For high school children, providing parents with materials with information about the utility of mathematics in STEM careers led to gains in parents' mathematics utility value, the number of STEM-related courses that children chose in their junior and senior years of high school, and engagement in STEM-related career fields. (Harackiewicz, Rozek, Hulleman, & Hyde, 2012; Rozek, Svoboda, Harackiewicz, Hulleman, & Hyde, 2017). However, this type of intervention has not been done with elementary school children. If parents are given helpful support, gains such as these may be possible for elementary age children as well.

## Limitations and future directions

Although the present study provided new information about how children define mathematics and how they see mathematics relating to their lives, there are several limitations to consider. The relatively small sample size prevented exploration of differences related to children's racial or ethnic group and parents' socioeconomic status and educational level. Relatedly, although Eccles et al. (1993) subjective task value scale, which was adapted for this study, was validated with a primarily European American/White sample, it has not been validated with ethnically diverse children. Another limitation is the timing of data collection. As noted, most of the children (82%) were interviewed during the summer months, when they typically have less exposure to academic instruction. This may have impacted how frequently they engaged in mathematics activities at home as well as their concepts of when, where, and by whom mathematics is used. Analyses comparing the responses of children interviewed in the summer with those in the fall showed a similar pattern of responses. And, most children's responses, regardless of the timing of data collection focused on school-related conceptions.

Perhaps the largest potential limitation is the inability to distinguish whether children lacked mathematics utility concepts or just could not articulate well their conceptions. Clearly, children's verbal abilities increase with age (Berko Gleason & Ratner, 2012; MacWhinney, 2010). However, children as young as preschool have demonstrated the ability to use rating scales and to describe self-concepts about their academic abilities and learning in reliable and valid ways. For example, researchers successfully used rating scales with children in preschool through Grade 6 to measure mathematics self-concepts (Marsh, Ellis, & Craven, 2002), subjective task value (Eccles et al., 1993; Wigfield et al., 1997), frequency of engagement in home mathematics activities (Ramani & Siegler, 2008), and child-teacher relationships (Li, Hughes, Kwok, & Hsu, 2012). The rating scales used in these research studies yielded acceptable to high reliability estimates with young children, were related to achievement and parent/teacher ratings, and showed consistent longitudinal patterns. Additionally, Mazzocco et al. (2012) asked children as young as Grade 2 to describe their definitions of mathematics. Children were able to provide responses that were related to their third grade mathematics achievement. These examples of child-reported mathematics self-concepts demonstrate young children's ability to report their beliefs about mathematics. Finally, the coding schemes used in this study as well as the length of utterance analyses controlled for potential developmental differences in length and sophistication of responses. Given the wealth of evidence showing that children are reliable reporters, we think our results are valid. Our methods also gave children a voice in research that may impact their education (Grover, 2004).

Future researchers should explore the relation between children's mathematics utility conceptions and mathematics achievement. Mazzocco et al. (2012) found significant positive associations between second grade children's definitions of mathematics and their number skills in third grade using the Calculation subtest from the Woodcock Johnson-Revised Tests of Achievement (Woodcock & Johnson, 1989). Additional research should focus on how children's mathematics utility conceptions relate to a broader array of mathematics skills (e.g., conceptual understanding, problem solving). Future researchers also should test the relations between the knowledge and belief about mathematics utility conceptions components of the conceptual model used in this study.

Additionally, research has not examined the potential effects of home and classroom interventions on children's mathematics conceptions. Documenting longitudinally how children's mathematics utility conceptions develop naturally over time at home and in school is important. Research needs to explore how parents' and teachers' mathematics utility conceptions impact the development of children's conceptions. Other home and school factors, such as amount of time children spend engaged in mathematics learning in the classroom, the frequency with which parents help their children with mathematics homework, or the extent to which teachers and parents label daily activities as "mathematics" may be associated with the development of children's mathematics utility conceptions. Such information could serve as the basis for interventions to improve children's mathematics utility conceptions.

## Conclusion

The primary goals of this study were to investigate children's mathematics utility conceptions and understand whether children's engagement in mathematics at home is associated with their mathematics utility conceptions. Exploring children's conceptions about how mathematics is used and by whom may help guide future interventions to improve mathematics learning. By increasing young children's knowledge of applications of mathematics outside the school context and beliefs about the usefulness of mathematics, parents and educators can help to increase children's mathematics proficiency.

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